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13. SUPPLEMENTARY NOTES

14. ABSTRACT This project has developed a group-theoretic approach to generate new tools of signal processing in order to analyze and track motion. For that purpose, external motion is projected on a sensor array and captured as a spatio-temporal signal to analyze. This research takes into account differential manifolds on which motion takes place and on which the sensor array is deployed. The research has derived the means to generate all the observable kinematics in a spatio-temporal signals. Each particular kinematic is defined by a specific Lie algebra. Each algebra leads to a Lie group, and to the appropriate group representations in the functional space of the signals. Theses representations lead to the related harmonic analysis. This motion analysis deals with translational, rotational, and deformational motion and all the temporal derivatives. This research has derived the optimal tools to estimate motion, to track moving patterns, to study diffusion and optical flow along motion trajectories. This group-theoretic framework covers determinist and stochastic calculus with Kalman filters, PDE's, ODE's and integral transforms. New algorithms to analyze and track motion have been defined from this theoretical framework and lead to parallelizable implementations based on FFT and dynamic programming.

15. SUBJECT TERMS
motion detection and estimation, target tracking, selective reconstruction, Lie group, Lie algebra, group representations, continuous and discrete wavelet transforms, harmonic analysis.

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Final Report

Grant F49620-99-1-0068
Title: Spatio-Temporal Wavelets for Motion Detection and Target Tracking
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1 Objectives

Spatio-temporal signal captured by an array of sensors like digital image sequences can provide useful inputs to a wide variety of automated systems, such as autonomous vehicle navigators, and surveillance systems to mention a few. The design of signal processing techniques that allow motion information to be reliably extracted remains a challenging problem, particularly when noise, jitter, and temporary occlusions are present. In this project, the PI has addressed the problem of motion analysis spaces from a new theoretical perspective based on Lie group representations to develop the harmonic analysis associated with Lie groups of spatio-temporal transformations. The entire construction of harmonic analysis is based on two assumptions related to the actual physics of motion:

1. locally, any observable motion transformation or kinematic is described by a Lie algebra or a Lie groups

2. globally, any trajectory is defined from a variational principle of optimality.

The harmonic analysis of motion transformations fundamentally differs from the other current modelings presented in the literature which mainly rely on techniques based on stochastic processes, statistics and operations research. As a major drawback, these existing techniques are totally blind to the underlying mathematical and kinematical structures of the spatio-temporal transformations. The research that has be done before 1995 to analyze motion with a group-theoretic point of view was only focusing on biological perception. In fact, biological sensors behave differently from electronic sensors.

The spatio-temporal transformations of concern deal with motions taking place in the outer space of observation. The tools to be developed analyze these spatio-temporal signals after their projections in the cone of visibility of sensor arrays (like video or infra-red cameras). In this work, the motion trajectories to analyze occur on smooth manifolds and the sensor arrays may be deployed on some other smooth manifolds. Three categories of motion are considered and referred to as translational, rotational or deformational. In each category, motion parameters are defined from all the temporal derivatives i.e. position, velocity and accelerations. Motion analysis means first of all selective detection, estimation and tracking. But it turns out that the continuous wavelets developed in this program present other efficient properties that able to cope with additional applications, namely spatio-temporal interpolation, selective motion-compensated filtering, signal decomposition, and selective reconstruction. An innovative study of selective motion tracking is based on the adjunction of variational principles of optimality to the group representations in order to define the appropriate equations of wavelet motion, the tracking equations, selective constants of motion to be tracked, and all the symmetries to be imposed on the analyzing wavelets. The study of the harmonic analysis associated with the new motion groups is expected to derive new tools to be introduced in tracking algorithms. This important issue leads to the definition of special functions, propagators and motion-compensated filters. Special functions provide in the Fourier domain specific spectral signatures

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based on specific motion to detect or estimate. Propagators generate any motion-tuned wavelet from their still cognate. Motion-compensated filters generate a still version from a moving pattern i.e. unwarp the motion transformation. Propagators and motion-compensated filters may as well process any moving pattern in order to synthesize a new version moving with different motion characteristics.

Nice properties developed from the modern abstract algebra apply to this theoretical framework to supply fast and highly parallelizable computational algorithms based on Fast Fourier Transforms for the continuous wavelet transforms and the inverse problem of reconstruction, on gradient algorithms for the minimum-mean-square-error estimation, and on dynamic programming techniques for tracking.

An example of application for this theoretical framework may be summarized as follows. Let us consider a passive system hosted in a mobile vehicle tracking selected moving targets with one single camera. First, the trajectory is decomposed into small charts spanning over a few seconds covering several hundreds of frames (typically 100 to 1000). Second, we assume that the surface on which motion transformations occur is known. In this case, the environment is acquired and described by a set of external surrounding active sensors (made of ground radars or satellites) that transmit the information to the algorithm that passively performs estimation and tracking. The information supplied about the target contains surface (geometry as curvature) and motion (initial or updated moving conditions). At the beginning of each chart, the passive tracking system receives from its base the update of the kinematics. From the initial kinematical conditions on the chart, the algorithm is capable to adjust its tracking and decides which wavelet family to use to keep on performing motion estimation and tracking until the next update.

2 Past accomplishments on the topic (not covered by AFOSR)

2.1 Period 11-1-93 to 2-1-96

During that period, the PI combined discrete spatio-temporal wavelet filtering with motion analysis algorithms based on operations research methods and statistics for image sequence processing. The PI developed the technique of motion-compensated filtering which means filtering along the assumed motion trajectories. The main problem in this approach was the huge computational load of the algorithms to estimate the motion parameters and compute the trajectories. The new idea was to design wavelet transforms as families of convolutional filters tuned to capture and extract objects in motion out of a scene. The wavelet should be continuous to provide a continuous range of motion parameter estimation. The first parameters to be considered were the scale, the spatio-temporal location, the orientation and the velocity. This refers to the so-called Galilei group known by physicists.

2.2 Period 5-1- 96 to 6-1-97

The purpose of this research work was to develop an application of the Galilean wavelet transforms to perform automatic target recognition. The particular problem addressed in this research was the tracking of a warhead for an incoming ballistic missile.

During that period, the PI has evidenced the MMSE property of the continuous wavelet transforms and their matching with Kalman filters. Dynamic programming algorithms have been used to implement the tracking algorithms. These algorithms were based on FFT's and optimal control with highly parallelizable schemes.

3 Final Status of Effort

During the initial period of this project, the PI started to investigate the existence of new continuous wavelet transforms for accelerated translational motion, for rotational motion (angular velocity and accelerations), and for deformational motion (temporal derivatives of the change of scale). For these transformations, the PI demonstrated the existence of Lie algebras, Lie groups, Lie group representations, and eventually the

existence of continuous wavelets for sets of motion parameters whose dimension is not larger than the dimension of the phase space (i.e. the Fourier space of translation and central parameters).

From these new findings, the PI kept on examining more general cases dealing with the analysis of motion on smooth manifolds. At this level of generalization, the PI evidenced the importance of extending and deforming Lie algebras: this enables to visit all the physically possible and admissible kinematics according to time and parity reversal. These kinematics are also those embedded in digital signals. Starting with admissible Lie algebras, the construction of the group composition is made possible using the Baker-Hausdorff formulas. The group representations are thereafter computed by the usual method developed by Mackey, Auslander, Pukansky and Kirillov. These group representations for motion on smooth manifolds remain subject to generalized square-integrability conditions. They give rise to generalized Fourier transforms to be considered as integral equations whose properties remain to be studied. Moreover, as curvature may vary from one area to the other, we may consider the Cartan geometry defining bundles of Lie algebras, kinematics, Lie groups, continuous wavelets and integral transforms.

The problem of the cone of visibility of an array of sensor is solved. It consists to relate Fourier transforms, group representations, Lie groups, and Lie algebras in the outer space $\mathbb{R}^3 \times \mathbb{R}$ with their cognates in the captured signals in $\mathbb{R}^2 \times \mathbb{R}$. The assumptions are as follows. First, the motion in the outer space is taking place on a known surface described by the Gaussian curvature. Second, the initial motion conditions in terms of object-sensor, velocity, accelerations are also supposed to be known. Third, the sensor array may be a plane or any another smooth manifold. Under these assumptions, it turns out to be possible to design continuous wavelets in the captured signals that estimate the actual motion parameters taking place in the outer space $\mathbb{R}^3 \times \mathbb{R}$.

The PI has started to re-examine the motion transformations in the context of irregular sampling. It is clear that patterns in motion undergo a sampling which is function of the relative position of the object with respect to the sensor. This sampling varies from image to image with a smooth geometric transformation can be considered as a warping operation on a band-limited function. This way of approaching the problem relates to Clark's theorem on frame reconstruction formula. The PI has already shown that rotational and deformational motions have there own sampling properties and theorems. It seems that this topic still requires some more investigations since it has never been explored thoroughly beyond the affine group. Our motion models deal with extensions of the Galilei group for electronic sensor arrays and the Lorentz group for biological sensors. This topic interests the coding applications, and more specifically, the optimal coding scheme for image sequence with areas of interest. Here, we assume that the viewer is focusing on one single moving target for which he requires the maximum amount of information/resolution and that the viewer is not interested by anything else in the content of that scene. Optimal coding scheme involves the following sequence: selectively unwarping motion, decorrelation, quantization, entropy coding. After transmission, the inverse operations are taking place in the decoder.

The uncertainty principle is usually understood as a Heisenberg inequality involving the wavelet functions. There are basically as many inequalities as there are non-commuting relations in the Lie algebra. This topic has never been explored in depth for a kinematical framework. Moreover, alternative approaches abound in this still growing literature. In fact, besides the non-commuting operator theory, this area of research can be examined along two main avenues: using a non-complex point of view already covered by numerous theorems or exploiting the powerful machinery of complex analysis. Let us mention that the study of important uncertainties between scale, orientation, position and velocity have never been investigated thoroughly despite the fact that they can be observed in image sequences. The derivation of related tight bounds/inequalities for motion estimation seems to be an important topic to be studied.

The PI has re-examined the continuous wavelet analysis in the framework of the mathematical radar and sonar theory. In fact, the generalized ambiguity function originally defined on the Heisenberg group is more

than a cross-correlation function but, in fact, a continuous wavelet transform where the unitary irreducible group representations are square-integrable. Proceeding in that vein, we reach statistical detection theory, Weiner theory, singularity expansions for target identification, Kalman filtering and hidden Markov models: we have already mentioned that the continuous wavelet transforms smoothly match to those topics. If we rephrase this approach, this project is deriving a theory for passive radars which is similar to that one developed for active microwave radars. In our case, the moving targets are analyzed from a signal passively picked up from natural radiations. Moreover, the transmission channel consists in a projection on the sensor array within the cone of visibility. Proceeding one step further, we reach the theory of array processing and beamforming which has not been yet studied with the point of view of group theory.

A pending issue of great interest that should require more attention in future research is the area of motion-selective reconstructions based on the inverse wavelet transform. To solve this problem, the properties of convergence of the inverse formula are of importance and are closely related to the image reconstruction technique using the localized phase in the Fourier domain. In fact, the importance of the phase has never been investigated so far to describe the motion content and never related to properties of weak or strong convergence of the inverse wavelet transform (i.e. the corresponding Calderon reproducing formula). We may also suppose that these properties depend on the motion under investigation and the characters of the group representations. However, it is well known that under a variety of conditions, the Fourier phase is sufficient for image representation. The earliest context in which the Fourier phase has been recognized was for the Fourier synthesis of crystallographic structures. Moreover, computational results and theoretical analysis indicate that constrained and unconstrained image reconstructions from local phase are more efficient than the reconstruction from global phase i.e. computer operations are reduced and rates of convergence are improved. Moreover, the local phase reconstruction is also implementable with fast algorithms using highly parallel architectures. This topic remains as a future path of research.

Another effort has been devoted to characterizing new questions that may be of importance for motion analysis:

1. develop the statistics of higher order, n-spectra and coherence function to study motion evolution.

2. detect the disruptions of stationarity when a motion in progress changes of kinematic, is occluded, ends or starts.

3. introduce group representation theory in array signal processing.

4. investigate how information theory and sampling theory specifically contribute to motion transformations.

5. search alternative means of deriving ODE's and PDE's as equation of motion. Although this topic can be addressed with the so-called Laplace-Baltrami operators. it seems to remain a fairly open issue on which the PI will interact with Prof. Leon Cohen at CUNY.

6. relate item 5 to build a bridge to PDE's that encompass the research work done for optical flow, and for diffusion processes.

7. examine the huge potentiality to create new analysis tools. Let us mention recent novelties like 'curvelets', 'ridgelets' that should come out in our area as 'trajelets'.

4 Accomplishments/New Findings

The PI has showed the necessity to relate group representations to one principle of optimality in order to thoroughly formulate a selective tracking problem. In this vein, the PI exploits variational principles of optimality to derive equations of wavelet motion in terms of Partial Differential Equations, to derive equations of tracking evolution in terms of Ordinary Differential Equations. The adjunction of a principle

of optimality enables to determine the selective constants of motion to be tracked and the symmetries to be imposed to the wavelets in order to track.

Lie group theory leads to harmonic analysis. In our case, we develop the harmonic analysis related to the motion transformations. The PI started recently to investigate the related special functions and their relations with the Green functions of the wavelet motion PDE's. This approach generates new integral equations with Green functions as kernel. The study of the properties of these kernel is highly relevant. The PI has rediscovered the motion-compensated filters developed during the period 11-1-93 and 2-1-96 from this recent research.

From spatio-temporal continuous wavelets, the PI has determined the existence of tight frames and orthonormal bases for translational motion. The rotational case is also under study. From these bases, the PI will derive the minimal expansions of the moving patterns along a trajectory.

The premise relations between stochastic calculus, stochastic processes, Lie algebras, Lie groups, and group representations have been established that introduce probability density on Lie generators or on group representation. Brownian motions or diffusion processes can also be driven by group representations and drift along a trajectory. Following this approach, the Ito integral can generalized in this case. This topic will be subject to much more attention since it will allow to treat on the same footing deterministic, jittering and diffusing trajectories.

At the present stage, the PI has developed a theoretical framework that spans from harmonic analysis of motion transformation to the differential geometry (the Cartan geometry) on which motion occur. This unified framework now reaches the present research edges of complex analysis, information theory, array processing, statistical signal processing, stochastic process. The research framework encompasses irregular sampling, radar theory, signal decomposition, inverse problem of image reconstruction. As a result of the group-theoretic foundation and the objective of developing a smooth theory, this theoretical framework may now be examined from various scientific points of view without any mismatches, gaps or discontinuities. These points of views are as follows: optimal control, filtering theory, differential geometry, functional analysis, classical and stochastic mechanics, relativity, coding and transmission. As a matter of fact, this approach addresses problems rooted in engineering, in mathematical physics, in mechanics, and in pure and applied mathematics.

The PI has also established smooth matching links between continuous wavelet transform and the group representation theory, on one hand, and the stochastic and deterministic Kalman theory on the other hand. This construction provides a broader interrelated view on Kalman filtering which can be stated in terms of operator theory, stochastic processes, estimation and filtering theory. Depending on the applications, this framework deals with the motion tracking, diffusion process, prediction, interpolation and reconstruction.

Computer routines have been developed on Matlab5 for the estimation and tracking of patterns moving with jittered transformations embedded in severe noisy pictures (results published on Journal and Conference papers). Comparisons have been presented with competing techniques based on subband decomposition and block matching that can not resolve the occlusion problem. The image sequences which have been treated so far contain translational and rotational velocities, and accelerations. Deformational motion has also been examined to measure the actual component of velocity orthogonal to the sensor plane i.e. the time before collision.

5 Interactions/Transitions

The collaboration with Professor Guido Weiss (Mathematics Department at Washington University in Saint Louis) has progressed along the following fundamental topics successively related to the Galilean group

and its generalizations: conditions of existence of continuous wavelets and convergence of the inverse transform. Several important related topics are going to be covered as frames, orthonormal bases i.e. discrete spatio-temporal wavelets, spatio-temporal convolutions, generalized Fourier transforms. It has then been continued on the convergence problem of the Calderon reproducing formula. This formula stands as the wavelet condition of admissibility. This condition defines the continuous wavelet transform as an isometry operator associated with a set of well known properties. The convergence of the reproducing formula can be strong or weak according to the group under investigation and so is the topology. This topic is supposed to provide a theoretical support on the issue of the motion selective reconstructions.

The collaboration with Professor Victor Wickerhauser (Mathematics Department at Washington University in Saint Louis) has covered several topics: reproducing kernels, Euler-Lagrange equations, square-integrability over the different motion groups of interest.

The collaboration with Professor Brian Blank (Mathematics Department at Washington University in Saint Louis) has focused on the construction of special functions related to the group representations, especially for rotational motion (trigonometric and hyperbolic rotations), and for deformational motion. Although being different, these special functions behave like Bessel functions.

Lively interactions with Professor John Benedetto (Mathematics Department at University of Maryland) have been initiated and will turn into a collaboration devoted on investigating the potential issues of interest that may not have been addressed so far in this research program: topics like the irregular sampling related to motion transformations and warpings, like the construction of new analytic expressions of discrete wavelet transforms associated with the group of motion under study, like new group representations in complex functional spaces.

Lively interactions have been realized with Professor Bijoy Ghosh. (Systems Science & Mathematics Department at Washington University in Saint Louis) Professor B. Ghosh is supporting all my heavy computational efforts. Discussions with Professor Bijoy Ghosh have dealt with the applications of motion estimation in robotics. Professor Ghosh has been very interested by the potentiality of having at hand new numerical tools that can estimate motion parameters like angular and deformation velocity. The fact of being able to estimate in captured signal the actual motion parameters taking place on known surfaces in the outer space seems to be of tremendous importance for robotic applications. The motion detection, estimation and tracking is one of the most important property of robotic vision must have. The efficiency of these estimation techniques will tremendously influence the autonomous capacity of a robot.

Lively interactions with Professor Lawrence Conlon (Mathematics Department at Washington University in Saint Louis) have been pursued during the past academic year about differential geometry. The purpose was to understand how geometers tackle the problem of generating and transporting bases on smooth manifolds and geodesics. For instance, how to properly define an orthonormal basis on some Riemannian geometry and how to process it in a parallel transportation on a geodesic. This subject is a generalized version of how to design discrete wavelets and how to displace them along the assumed trajectory of motion. Both 'top down' (by the differential geometry) and 'bottom up' (by the wavelet theory) approaches are expected to match.

Lively interactions with Professor Renato Feres (Mathematics Department at Washington University in Saint Louis) have help to develop the means to introduce noise generators in the Lie theory. Noise and random processes can be introduced in the algebra, the group and in the group representations in order to yield the appropriate stochastic calculus that combines diffusion process and motion transformation. Restated in ather words, problem at hand consisted on how to model a diffusion process on a motion trajectory. This research effort leads towards stochastic PDE's and integral equations of the Ito family. This topics of combining stochastic processes and group theory has also been a topic of interest in the Department of Electrical

Engineering at Washington University in Saint Louis where Professor J. O'Sullivan is our interlocutor.

Lively interactions have been initiated with Professor Welland (Mathematics Department of the University of Missouri at Saint Louis) on alternative ways of tackling the problem of motion analysis. We adopted the perspective of biological and human perception properties. Biological sensors behave differently from the electronic sensor arrays. Since biological sensors rescale time and space, we are conducted to considering other Lie groups based on the Lorentz transformations. In fact, this biological approach of motion analysis based on the Human Visual System has been a topic studied with group representations before the author started this research program. Among these previous works, let us mention authors like Caeli (1978), Zeevi (1992), Duval-Destin and Murenzi (1992).

Lively interactions with Professor Mohan Kumar (Mathematics Department at Washington University in Saint Louis) have covered the topic of how abstract algebra can help to develop fast algorithms. It is well known that the Fast Fourier Transform developed by Cooley-Tucker relies on algebraic structures like the Chinese-remainders theorem, the Poisson formula and some properties of the group characters. The polynomial ring theory is central to many applications at hand for both multidimensional convolution and multidimensional Fourier transform. This approach is related to the H. J. Nussbaumer's work where polynomial transforms use the Chinese remainders to decompose the global computation into subcomputations. This discussion is primordial to be aware of how fast algorithms may be implemented, and therefore, to be aware of how to set up appropriate structures and ideas that lead to algorithms of estimation, tracking and reconstruction allowing fast parallel computations.

During the period 9-1-97 to 5-1-99, the PI ideas have supported the writing of a Ph.D. dissertation for a recipient student in the Mathematics Department of Washington University in Saint Louis. It was an actual opportunity for this Ph.D. student, Jon Corbett, to learn the theory of group representations using Mackey's theory and Kirrilov's technique of orbits, to learn the mathematical physics concerning the Galilei and Poincare groups as well as the properties of the continuous wavelets over several groups of motion transformation. The Ph.D. dissertation of Dr. Jon Corbett is provided in annex of this final report.

During the period 9-1-96 to 9-1-97, the PI has developed an application of the continuous wavelet transforms that performs automatic target recognition co-advising a graduate student F. Mujica at the Georgia Institute of Technology. The particular problem that has been addressed in this research is the tracking a warhead in an incoming ballistic missile. The scenario assumes that the ballistic missile has fragmented and that the wearhead is among the missile fragments. The task at hand is to develop a robust algorithm that will allow the interceptor to track and neutralize the threat. An optical sensor housed in the interceptor's nose cone provides the images that are processed and used to direct the course of interception. The main practical focus in this application is the selective tracking of small moving objects embedded in digital image sequences and the construction of their trajectories. The digital signals that originate from the sensors are sampled at a rate of 100 images per second. Realistic assumptions have been considered for motion and noisy data. The motion to be tracked is not only linear and uniformly accelerated but also jittered by a random motion on the sensors (Gaussian noise of standard deviation of one pixel away from the trajectory). The image quality can also degrade quite significantly to display poor signal-to-noise ratio less than 10dB signal-to-noise ratio. The algorithm to be developed should work as a framework associating continuous wavelet transforms and signal representations with other efficient techniques in order to perform motion and velocity detection, measurements or estimations and selective tracking. The Ph.D. dissertation of Dr. Fernando Mujica can be obtained at library of the Georgia Institute of Technology under the title: "Spatio-temporal continuous wavelet transform for motion estimation", 1999.

During the period 9-1-97 to 5-1-99, the PI ideas on how to apply the Galilean wavelets to perform motion-based segmentations of digital scenes have been implemented by Mingqi Kong a Ph.D. student in the Systems-Sciences-Mathematics Department of Washington University in Saint Louis. As expected

the continuous wavelet transform is efficient to sketch all the spatio-temporal edges or discontinuities i.e. the tubes of motion. In this approach, we get on the same footing the edges and the estimation of the motion parameters (this is not the case in classical techniques where edges are required to estimate motion and vice versa). This approach may then successfully simplify the usual algorithms based on operation research techniques specially for target pursuit. The Ph.D. dissertation of Mingqi Kong can be found on the dissertation abstract on line and ordered at the library of Washington University in Saint Louis under the title; "Motion Estimation and Motion-Based Segmentation in Digital Image Sequences", 1999.

Paper presentations at Conferences and Seminars:

Presentation of seminars after invitation in the department of electrical engineering at the New Jersey Institute of Technology (New Jersey, 1996), Polytechnic Institute of Brooklyn (New York, 1997), Rochester University (New York state, 1996), Rensselear Polytechnic Institute (New York state, 1998), University of Texas, San Antonio (Texas, 1997).

Presentation of seminars after invitation in the department of sciences, systems and mathematics at the Washington University in Saint Louis (Missouri, 1998).

Presentation of seminars after invitation in the department of mathematics at the Texas A&M University, College Station (Texas, 1996), University of Maryland at College Park (Maryland, 1996), Rensselear Polytechnic Institute (New York state, 1998), University of Missouri in Saint Louis (Missouri, 2000).

The research has also been presented at the "Institut de Recherche en Informatique et Systèmes Aléatoires (IRISA, Campus de Beaulieu, Rennes, France): "Estimation et pistage du mouvement par ondelettes continues spatio-temporelles", 1995.
1. ICIP-98, Chicago, October 5, 1998 (two papers).

2. Time-Frequency and Time-Scale Analysis: Pittsburgh, October 6, 1998 (1 paper).

3. IT Workshop on Detection, Estimation, Classification and Imaging: Santa Fe,
February 26, 1999 (1 paper).

4. ICASSP-99, Phoenix, March 16, 1999 (1 paper).

5. two presentations in the Mathematics department on Lie group representation theory: "Properties of Coherent States and Continuous Wavelets", 1997, and "Group Representation Theory for Nilpotent, Solvable and Exponential-Solvable Groups", 1998.

6. one presentation in the department of Systems Science and Mathematics at Washington University in Saint Louis about the PI on-going research.
7. ICASSP-2000, Istanbul, Turkey, June 5-9 2000 (1 paper).

8. NATO-ASI-Conference: 20th Century Harmonic Analysis, A Celebration, Il Ciocco, Italy, July 2-15 2000 (1 poster).

9. IEEE Workshop on Statistical Signal and Array Processing Pocono Manor, Pennsylvania, August 14-16 2000 (1 paper).

10. IEEE International Symposium on Information Theory, Sorrento, Italy, 25-30 June 2000 (1 paper).

11. invited presentation in the department of Electrical Engineering at Washington University in Saint Louis.

12. presentation at the Wavelet and Harmonic Analysis seminar of the Mathematics department of the University of Maryland : "Group-Theoretic Approach for the Harmonic Analysis of Spatio-Temporal Transformations", December 2000.

13. IEEE International Symposium on Information Theory, Washington DC, 24-29 June 2001 (1 paper).

6 Further Paths for Research

The further research will focus on developing a unified theoretical framework that deals with harmonic analysis associated with motion analysis. We shall derive *new mathematical tools* to process spatio-temporal digital signals, *new algorithms* for optimal motion estimation and tracking, and *new fast parallelizable hardware implementations* similar to the Cooley-Tukey algebraic structures or the quantum Fourier transform. The first principle is to model *local* motion transformations by *Lie algebras*. Lie algebras characterize both smooth manifolds and motion in an way which is easy to handle. The construction starts with Lie algebras and proceeds to Lie groups, and then to group representations on function spaces, e.g., Hilbert spaces are often appropriate for signal analysis. This construction establishes a connection between motion on differentiable manifolds and operators acting on functions. Some properties of these operators allow us to derive motion-based correlators, as well as continuous and discrete wavelet transforms required for effective motion analysis. This is the foundation of our entire theoretical framework.

Our mathematical approach will incorporate the physical nature of motion in digital signal analysis, and in human visual perception. Further, a *variational principle of optimality*, based on Lagrangians or Hamiltonians, is required to link all the local transformations into trajectories. This provides tracking abilities and trajectory constructions with fast algorithms from dynamic programming. This theoretical framework generalizes all the current techniques of motion and diffusion analysis presented in the literature. Bayesian and non-Bayesian statistical inferences have to be adjoined to our Lie-theoretic construction for optimal estimation and tracking purposes.

Consider motion taking place in $\mathbb{R}^3 \times \mathbb{R}$ (3-dimensional space and time). In this case, for the purpose of analysis, motion is projected onto sensor arrays and captured as apparent motion in $\mathbb{R}^2 \times \mathbb{R}$ digital signals or in the human visual system. The project investigates motion and sensors deployed on smooth manifolds, and the projective transformations that relate them. The set of motion transformations under consideration contains translation, rotations, and dilation along with their temporal derivatives, i.e., velocity and acceleration. Stochastic processes can be introduced at different levels of the theory to consider stochastic geometry, random array sensors, jittered motion, and diffusion processes.

The construction of this unified framework requires the following research.

1. Generate new *Lie algebras, Lie groups, group representations, and discrete and continuous wavelet transforms*, and introduce stochastic processes at the appropriate stages. These new algebras are necessary to treat motion on smooth manifolds associated with tracking and estimation for applications dealing with non-planar sensor arrays.

2. Investigate the properties of the *group characters* and design fast parallelizable hardware implementation of the special functions associated with these characters.

3. Derive *the uncertainty principle* required to obtain the optimal strategy for estimation and tracking.

4. Exploit *operator theory, stochastic calculus, and optimal control* in order to generate new sets of ODEs and PDEs that govern selective tracking and diffusion processes. Formulate new orthogonal bases and convolutional motion-compensated filters. These new models will determine equations of the selective tracking along with their corresponding tools of analysis.

5. Investigate the *irregular sampling problem* related to random array sensors, the human visual system, and motion-selective reconstruction.

6. Develop the theoretical framework with *models, a DSP toolbox, and fast parallelizable algorithms* to treat *motion analysis in spatio-temporal digital signals* and in the *human visual system.*

To outline the main orientations of this theoretical research, we can see that the group-theoretic approach remarkably provides all the appropriate analytical formulations related to motion analysis and diffusion process, the whole support to perform optimal tracking (uncertainties, correlators, estimators, PDE's and ODE's), and the structure of the fast algorithms for motion estimation and reconstruction.

7 Conclusions

8 Publications

8.1 Publications in Conferences

1. [17] J.-P. Leduc, J.-M. Odobez, C. Labit, *"Adaptive Motion-Compensated Wavelet Filtering for Image Sequence Processing"*, ICASSP-95, Detroit, USA, 8-12 May 1995, Vol. 4, pp. 2367-2370.

2. J.-P. Leduc and C. Labit, *"Spatio-Temporal Wavelet Transforms for Image Sequence Analysis"*, VIII European Signal Processing Conference, EUSIPCO-96, Trieste, Italy, 10-13 September 1996, 4 pp.

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4. F. Mujica, R. Murenzi, M. Smith and J.-P. Leduc *"A New Object Tracking Algorithm Based on The Continuous Wavelet Transform"*, SPIE Journal of Electronic Imaging, pp. 746-754, Vol. 7, October 1998.

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9 Annexes

Additional pieces of information to support the understanding and the transfer of research related to this project are provided as follows in 6 annexes:

1. the group-theoretic framework is outlined in annexe 1.

2. the Ph.D. thesis dissertation in mathematics of Dr. Jonathan Corbett, a more precise and intimately related material can be find in the following thesis of C. Kalisa "Etats Cohérents Affines: Canoniques, Galiléens et Relativistes",Ph.D. thesis, Université Catholique de Louvain, 1993. These theses provide similar computation of Lie group representations associated to computations of kinematical groups like the Galilei group.

3. a paper under review dealing with accelerated wavelets.

4. a paper under review dealing with rotational wavelets.

5. demonstration routines on Matlab5. Most of the explanations on how to deal with those algorithms is provided in two published papers: (1) sf F. Mujica, J.-P. Leduc, R. Murenzi, and M. Smith "A New Parameter Estimation Algorithm Based on the Continuous Wavelet Transform", IEEE Transactions on Image Processing, Vol. 9, No. 5, May 2000, pp. 873-888, and in paper (2) F. Mujica, R. Murenzi, M. Smith and J.-P. Leduc "A New Object Tracking Algorithm Based on The Continuous Wavelet Transform", SPIE Journal of Electronic Imaging, pp. 746-754, Vol. 7, October 1998

6. additional helpful references from the literature